locities and practically the most interesting, belongs to pressure drops dependent on gas velocity, liquid flow rate, and liquid shear properties. For this region, it was intended in the present work to construct the design correlation for gas pressure drop.

In Figures 1 and 2, the experimental data on gas pressure drop dependence on gas velocity and liquid flow rate were shown, respectively.

In Figure 3, it is shown that the gas pressure drop in the two phase flow of pseudoplastic films and gas streams highly depends on liquid rheological properties.

Consequently, the following design correlation has been constructed:

$$\Delta P = 0.0438\, v_g^{2.397}\, Q^{0.206/n}\, K^{0.172} \qquad (2)$$

Comparison of Equation (2) with experimental data shows the following standard deviations:

Water	7.11%
Liquid 1	7.80%
Liquid 2	4.92%
Liquid 3	12.62%

Of course, an extensive experimental work for gas velocities from 0 to 40 m/s should be conducted to study all the regions of gas pressure drops. In any case, this work presents the correlation which covers practically the most important region of gas pressure drop in concurrent or pseudoplastic liquid films and gas streams.

NOTATION

K = consistency factor, dyne sec^n/cm^2
L = length of plate, cm
n = flow behavior index
Q = liquid volume flow rate per unit width of the plate, cm^3/cm s
v_g = gas velocity, cm/s
γ = rate of shear, s^{-1}
τ = shear stress, dyne/cm^2
P = pressure drop, mm water

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Critical Reynolds Numbers for Newtonian Flow in Concentric Annuli

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A number of years ago, a transition parameter was developed to compute critical Reynolds numbers of transition from laminar to nonlaminar flow in straight-walled ducts of arbitrary cross-section (Hanks 1963). This parameter is practical and useful for engineering design purposes, because with it, Re_c can be accurately predicted, regardless of duct geometry or fluid rheology.

Originally, Hanks (1963) applied this parameter to the case of Newtonian flow in concentric annuli. A curve of Re_c is presented, based on a special equivalent diameter defined by Lohrenz and Kurata (1960), as a function of annular aspect ratio ($\sigma = R_i/R_o$). In a more detailed study of transitional flow phenomena in concentric annuli Hanks and Bonner (1971) analyze the annular flow field in terms of two separate regions: 1) an inner or core region defined by $\sigma \leq \xi \leq \lambda$; and 2) an outer or wall region defined by $\lambda \leq \xi \leq 1$, where $\xi = r/R_o$ and $\xi = \lambda$ where the velocity is a maximum. By developing these separate velocity profile expressions and applying the transition parameter to each region separately, Hanks and Bonner showed that a second critical Re, different from the one originally calculated by Hanks (1963) could be predicted. They also showed that Hanks' original calculations corresponded to their results for the outer or wall region of the flow.

This note will show that division of the annular flow into two regions for transitional flow analysis is artificial, and that upon more careful analysis, Hanks' original theory can be shown to predict both of the Re_c values found by Hanks and Bonner in 1971.

THEORETICAL ANALYSIS

The transition parameter (Hanks 1963) for this problem takes the form

$$K = \frac{\rho \,|\, \nabla(\tfrac{1}{2}v^2) \,|}{|\, \mathbf{f} - \nabla p \,|} \qquad (1)$$

where $\mathbf{f}$ is the body force, p is the pressure and $v^2 = \mathbf{v}\cdot\mathbf{v}$. For Newtonian flow in a concentric annulus the velocity distribution is well-known (Bird et al. 1960) to be

$$v(\xi) = \frac{R_o^2}{4\mu}\left(\frac{-dp}{dz}\right)[1 - \xi^2 + 2\lambda^2 \ln\xi] \qquad (2)$$

where $\xi = r/R_o$ and $\lambda^2 = (\sigma^2 - 1)/\ln\sigma^2$. Lohrenz and Kurata (1960) define an equivalent diameter $D_e = 2R_o\sqrt{\phi(\sigma)}$ with $\phi(\sigma) = 1 + \sigma^2 - 2\lambda^2$. When one uses D_e in defining the Fanning friction factor, $f = D_e(-dp/dz)/2\rho\langle v\rangle^2$, and $Re = D_e\langle v\rangle\rho/\mu$, these variables will satisfy the simple relation $f = 16/Re$.

By combining the above results, it is simple to express Equation (1) in the form

TABLE 1. THEORETICAL NUMERICAL VALUES OF $\bar{\xi}_1$, $\bar{\xi}_2$, Re_{ic} AND Re_{oc} FOR NEWTONIAN FLOW IN CONCENTRIC ANNULI

σ	$\bar{\xi}_1$	$\bar{\xi}_2$	Re_{ic}	Re_{oc}
0	0	0.5774	0	2,100
0.02	0.05161	0.6982	893	2,396
0.05	0.1137	0.7233	1,246	2,433
0.1	0.1915	0.7510	1,517	2,455
0.2	0.3107	0.7919	1,783	2,459
0.3	0.4113	0.8254	1,930	2,445
0.4	0.5037	0.8552	2,029	2,425
0.5	0.5915	0.8827	2,100	2,402
0.6	0.6764	0.9084	2,155	2,378
0.7	0.7592	0.9328	2,198	2,354
0.8	0.8406	0.9561	2,233	2,331
0.9	0.9207	0.9785	2,261	2,308
1	1	1	2,285	2,285

$$K = \frac{Re}{2\phi^{3/2}} (1 - \xi^2 + 2\lambda^2 \ln\xi) \left| \xi - \frac{\lambda^2}{\xi} \right| \qquad (3)$$

The critical Reynolds number is computed by setting $K = 404$ in Equation (3) (Hanks 1963) and introducing $\xi = \bar{\xi}$, where $\bar{\xi}$ is the location within the cross-section where $dK/d\xi = 0$; that is, where K exhibits its maxima. Since $K = 0$ at $\xi = \sigma, \lambda, 1$ and $K \geqq 0$ exerywhere (Hanks 1963), it follows that two maxima exist in the $K(\xi)$ curve in the range $\sigma \leqq \xi \leqq 1$.

If we follow the suggestion of Hanks and Bonner (1971) and consider the two regions $\sigma \leqq \xi \leqq \lambda$ and $\lambda \leqq \xi \leqq 1$ separately, we obtain the following results:

Inner Region $(\sigma \leqq \xi \leqq \lambda)$:

$$\left.\frac{dK}{d\xi}\right|_{\bar{\xi}_1} = 0 = 2(\lambda^2 - \bar{\xi}_1{}^2)^2 - (1 - \bar{\xi}_1{}^2 + 2\lambda^2 \ln \bar{\xi}_1)$$

Outer Region $(\lambda < \xi \leqq 1)$:

$$\cdot\, (\lambda^2 + \bar{\xi}_1{}^2) \qquad (4)$$

$$\left.\frac{dK}{d\xi}\right|_{\bar{\xi}_2} = 0 = (1 - \bar{\xi}_2{}^2 + 2\lambda^2 \ln \bar{\xi}_2)(\bar{\xi}_2{}^2 + \lambda^2) - 2(\bar{\xi}_2{}^2 - \lambda^2)^2 \qquad (5)$$

It is easily shown that Equation (4) and (5) are identical in form except for a trivial factor of -1. Either Equation (4) or Equation (5), and Equation (5) in particular, contains both $\bar{\xi}_1$ and $\bar{\xi}_2$ as roots. Equation (5) is equivalent to Hanks' 1963 result, and hence we conclude that the original theory contained both roots, although at that time only $\bar{\xi}_2$ was recognized and calculated.

Equation (3) may be solved for Re_c by setting $K = 404$ to give

$$Re_c = \frac{808\, \phi^{3/2}}{(1 - \bar{\xi}^2 + 2\lambda^2 \ln \bar{\xi}) \left| \bar{\xi} - \frac{\lambda^2}{\bar{\xi}} \right|} \qquad (6)$$

Table 1 contains values of $\bar{\xi}_1$ and $\bar{\xi}_2$ calculated from Equation (5) for various values of σ. Also listed are the corresponding values of Re_{ic} and Re_{oc}, obtained when $\bar{\xi} = \bar{\xi}_1$ and $\bar{\xi} = \bar{\xi}_2$, respectively, are introduced into Equation (6). Note that these values differ numerically from those presented by Hanks and Bonner (1971), because Re as used here is defined in terms of D_e, which is different from their definition of Re. In actual fact, the values are precisely equivalent. These values are also displayed graphically in Figure 1 in comparison with experimental values estimated from the 1971 data of Hanks and Bonner. The

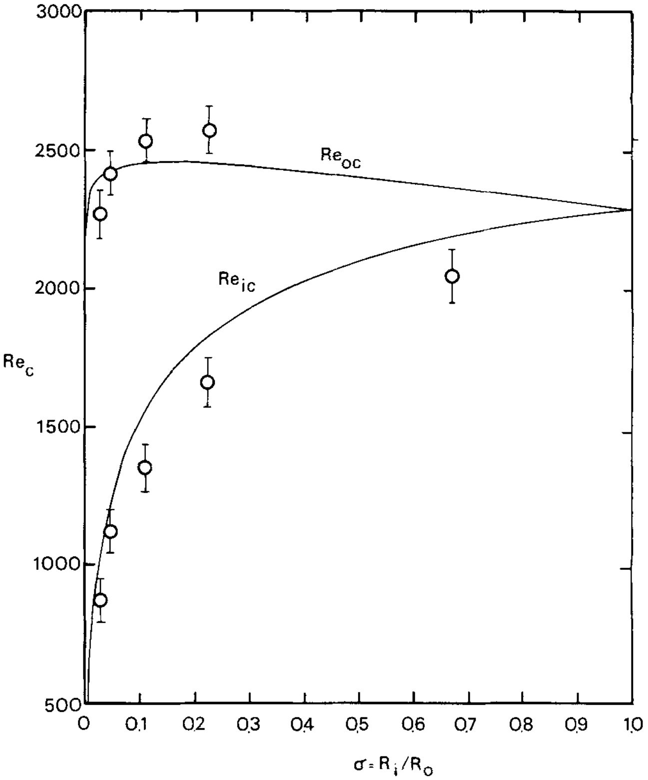

Figure 1. Re_{ic} and Re_{oc} as functions of $\sigma = R_i/R_o$ for Newtonian flow in concentric annuli. Limiting value for $\sigma = 1$ is $Re_{ic} = Re_{oc} = 2{,}285$. Data are estimated from curves of Hanks and Bonner (1971). Error bars indicate approximate range of uncertainty in data.

limit $Re_{ic} = Re_{oc} = 2{,}285$ at $\sigma = 1$ was shown previously by Hanks (1963) to be the limiting value obtained from the above equations.

DISCUSSION

From the results presented here, it is clear that the division of the annular flow field into an inner and an outer region for consideration of transition critical Reynolds numbers, as done by Hanks and Bonner (1971), is artificial and unnecessary. All the pertinent information is contained in Equations (2) and (3).

The data points in Figure 1, estimated from the 1971 data, clearly show the trends predicted by the theory. Hanks and Bonner argued that possible eccentricities of core resulted in the slightly early transitions for the first point (low Re_{ic} values). They substantiated their argument with data obtained in an annulus with a fully eccentric core. Thus, the agreement between the data and the Re_{ic} curve in Figure 1 is considered to be acceptably good. The Re_{oc} data points reflect the fact that some as yet not fully understood phenomenon occurs between Re_{ic} and Re_{oc}, which causes the Re_{oc} theoretical curve to be conservatively low.

NOTATION

$-dp/dz$ = pressure gradient in annulus
D_e = equivalent diameter, $2R_o\phi^{1/2}(\sigma)$
f = friction factor, $D_e(-dp/dz)/2\rho\langle v\rangle^2$
$\mathbf{f}$ = body force in equations of motion
K = transition parameter, Equation (1)
p = pressure
r = radial position variable
R_i = radius of inner core wall of annulus
R_o = radius of outer pipe wall of annulus
Re = Reynolds number, $D_e\langle v\rangle\rho/\mu$

Re_c = critical value of Re
Re_{ic} = Re_c for $\bar{\xi} = \bar{\xi}_1$
Re_{oc} = Re_c for $\bar{\xi} = \bar{\xi}_2$
v = axial velocity component
$\langle v \rangle$ = average of v
$\mathbf{v}$ = velocity vector

Greek Symbols

∇ = gradient operator
λ = value of ξ for which velocity is maximum
μ = viscosity
ξ = r/R_o
$\bar{\xi}$ = value of ξ at which K is maximum
$\bar{\xi}_1$ = value of $\bar{\xi}$ for $\sigma \leq \bar{\xi}_1 \leq \lambda$
$\bar{\xi}_2$ = value of $\bar{\xi}$ for $\lambda \leq \bar{\xi}_2 \leq 1$
ρ = density
σ = annulus aspect ratio, R_i/R_o
ϕ = $1 + \sigma^2 - 2\lambda^2$

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Diffusion Coefficients for Helium, Hydrogen, and Carbon Dioxide in Water at 25°C.

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Accurate values of the diffusivities of gases in liquids are useful in many different branches of research. Gas diffusivity data for sparingly soluble gases is of special interest since they are useful in testing the various theories of liquid phase diffusion for the case when the solute is of low molecular weight. In engineering, they are employed in investigations concerning the role of diffusion in interfacial mass transfer and as an aid in interpreting laboratory investigations, as well as for the design of mass transfer equipment. Moreover, they are frequently needed to interpret physiological studies.

A number of investigators have determined diffusivities of sparingly soluble gases such as helium and hydrogen in water, but because of a wide range of these values as shown in Table 2, their results are in need of confirmation. The main problem involved in these studies has been the difficulties associated with the accurate determination of trace quantities of dissolved gas.

The objective of this work was to determine the diffusion coefficients of two important slightly soluble gases, helium and hydrogen, in water. The method employed in this work requires no empirically determined parameters and has the advantage of being relatively fast. As a test of the reliability of this method, experiments were also carried out with carbon dioxide, a gas having a well-known diffusion coefficient. A wetted wall column technique was used with a relatively long (0.200 m) stainless steel tube as the wetted wall. Water was mixed with a small amount of a surface active agent to suppress surface ripples and give well-defined hydrodynamics. The apparatus and technique used were those employed by Perez and Sandall (1973) for diffusivity measurements in non-Newtonian liquids.

THEORY

A solution for the differential equation describing gas absorption into a liquid flowing in a falling film with a parabolic velocity profile was obtained by Pigford (1941). As a result of later research carried out by Emmert and Pigford (1954), it was concluded that for short contact times this solution reduces to the penetration theory prediction of Higbie (1935) which is based on the assumption of a uniform velocity.

Thus, for the short contact times of our experiments, an equation may be derived (Perez and Sandall, 1973) from penetration theory to relate the diffusivity to variables which can be measured or calculated:

$$D = \frac{\pi}{4} \frac{q^2}{U_s l} \left\{ \frac{r}{r+d} \right\}^2 \left[\frac{C_2 - C_1}{(C_s - C)_{lm}} \right]^2 \quad (1)$$

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus was that used by Lamourelle and Sandall (1972) and is shown schematically in Figure 1. The apparatus consisted of absorption and desorption sections so that the water could be recycled. A laminar film of water was formed on the outside of a 0.0159 m diameter stainless steel tube. The length of the wetted wall was 0.200 m. Flow rates were determined using a rotameter, and the water temperature was measured by a chromel-constantan thermocouple. In the course of the experiment, a mixture of distilled water and 0.3 wt % Petrowet R was used. Petrowet R is a surface active agent manufactured by the Dupont Company.

The solution had to be stripped of the gas in the desorption section after every run so that the water could be recycled. The stripping process was carried out at an absolute pressure of 1.6×10^4 pascals which was created by a filter pump.